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QUALITATIVE PROPERTIES AND ANALYSIS SOLUTION OF THE BOUNDARY VALUE PROBLEM FOR A DIFFERENTIAL EQUATION WITH IMPULSIVE ACTION

ABSTRACT

of the thesis for the degree of Doctor of Philosophy (PhD)
in the specialty 8D05401 – Mathematics

General description of the dissertation work. The dissertation work is devoted to the study of the solvability and qualitative properties of the boundary value problem for the ordinary differential equations with the impulse action at fixed and non-fixed times.

The relevance of the study is due to the numerous applications of the differential equations with the impulse action in solving natural science problems, as well as the importance of the develop new constructive methods that make it possible to effectively determine the solvability of boundary value problems for the differential equations with the impulse and to find their solutions.

The differential equations with the impulse effects serve as mathematical models of many objects that, in the process of their evolution, are subject to the action of short-term forces, for example, in nonlinear mechanics, in an automatic control, in the theory of oscillations and dynamic systems, etc. The presence of the impulse effects can significantly complicate the behavior of the trajectories of such systems even for the case of the simple differential equations, it is sufficient. The impulsive systems of the various types were considered in the classical works of N.N. Bogolyubov, N.M. Krylov, E.A. Barbashin, A. Khalanay, D. Veksler, A.D. Myshkis, A.M. Samoilenko, N.A. Perestyuk, D. D. Bainov, etc. The subsequent researches by many authors were devoted to the study of the stability of solutions to differential equations with the impulse action, the development of the theory of periodic and almost periodic solutions of impulsive systems, the study of invariant sets, the construction of asymptotic expansions using the Krylov-Bogolyubov-Mitropolsky small parameter method, averaging method, problems of optimal control theory, study of the impulsive systems with the random perturbations.

It is well known that the studies of real processes based on the idealized mathematical models often lead to the differential equations with the small parameters. Various asymptotic methods are widely used to study them. The choice of a specific asymptotic method depends on the structure of the differential equation describing the dynamics of the object. Due to the numerous applications, averaging methods have been widely developed in nonlinear mechanics and oscillation theory. The mathematical justification of the averaging method for the ordinary differential equations originates in the fundamental work of N.M. Krylov and N.N. Bogolyubov. A major role in the development of the averaging method for various classes of

differential equations had the works of E.A. Grebenikov, Yu.A. Mitropolsky, N.N. Moiseev, N.A. Perestyuk, V.A. Plotnikov, A.M. Samoilenko, A.N. Filatov and others.

The generalization of the averaging method for the asymptotic integration of impulsive differential equations is of great theoretical and practical importance for the following reasons: due to the complex structure of the impulse systems, qualitative research is associated with great difficulties, while the averaged system becomes without impulse; the solution of the averaged system approximates the solution of the original system with any predetermined accuracy on an asymptotically large time interval.

In this dissertation, the averaging method is developed for initial and boundary value problems for nonlinear differential equations with the impulse effects at non-fixed moments and depending on a small parameter.

Differential equations with the impulse actions are studied at non-fixed moments with a small parameter in the nonlinear right-hand side of the equation and under impulse conditions, where the boundary conditions are also specified nonlinearly.

The main methods for solving these classes of problems are the averaging method and the Dzhumabaev's parameterization method.

D.S. Dzhumabaev offered a new approach based on the parameterization method for studying and solving boundary value problems for the differential equations with the impulse action.

The aim of the thesis research. The application of the averaging and the parameterization methods in solving boundary value problems for the impulse differential equations and creation of effective algorithms for the solving problems.

The research problems:

a) the determination of the continuous dependence of the solution of the system of the differential equations of the impulse effects on the initial conditions;
b) the solving the equation of the variation of the impulse system using the averaging method;
c) the establish the conditions for the solvability of a boundary value problem for a differential equation with impulse action at a fixed time using the averaging method;
d) the obtaining conditions for solving a nonlinear boundary value problem for a differential equation with impulse action and creating an effective algorithm for searching for its solution;
e) the determination of conditions for the solvability of a boundary value problem for a differential equation with momentum at a non-fixed moment of time using the parameterization method;
f) establishing conditions for the existence of two-sided solutions of a differential equation with the impulse action bounded along the axis at a non-fixed moment in time.

The research methods. In the dissertation work, the method of averaging the solution of the boundary value problem and the method of parameterizing the solution of the boundary value problem were used.

The objects of research are a boundary value problem for a nonlinear ordinary differential equation with the impulse action at a fixed point in time, boundary value problem for an ordinary differential equation with impulse action at a non-fixed point in time.

The scientific novelties:

a) conditions for the existence of a solution to the boundary value problem for a differential equation with impulse action at a fixed point in time are established in the case of the existence of a solution to the averaged boundary value problem for the impulse differential equation;

b) the averaging method determines the conditions for the existence of two-sided solutions of a differential equation with the impulse action bounded along the axis at a non-fixed moment in time;

c) parameterization method D.S. Dzhumabaev was used to solve boundary value problems for a nonlinear ordinary differential equation with the impulse action at a fixed point in time;

d) parameterization method D.S. Dzhumabaev was used to solve a boundary value problem for a differential equation with the impulse action at a non-fixed point in time.

e) algorithms for solving a nonlinear boundary value problem and their numerical implementation for a differential equation with the impulse action at a non-fixed point in time have been developed.

The results of the thesis which are taken out on defense:

a) finding a solution to the equations of variation of the pulsed system using the averaging method;

b) application of the averaging method to study the existence of solutions to a boundary value problem for differential equations with the impulse action at fixed and non-fixed moments of time;

c) determination of two-sided solutions of a differential equation with the impulse action limited along the axis at an unfixed moment in time by the averaging method;

d) conditions for solving a nonlinear boundary value problem for a differential equation with impulse action at a fixed point in time;

f) solving a nonlinear boundary value problem for a differential equation with the impulse action using the parameterization method;

g) algorithms for solving a nonlinear boundary value problem for a differential equation with the impulse action at a non-fixed point in time and their numerical implementation;

h) an algorithm for searching for a solution to a system of nonlinear algebraic equations with respect to the parameters of the boundary value problem for a differential equation with the impulse action.

Reliability and validity. The dissertation makes extensive use of the methods and results of the theory of impulse differential equations. The main methods of research and solution of problems considered in the dissertation are averaging and parameterization methods.

Theoretical and practical significance of the research. The results of the dissertation work are mainly theoretical in nature. The scientific significance of the work lies in the creation of a constructive method for studying and solving boundary value problems for differential equations with the impulse action at non-fixed moments of time.

Connection of the dissertation work with other research works. The dissertation work was carried out within the framework of grant funding under the priority “Fundamental Research in the Field of Natural Sciences” within the framework of the project “Methods for solving boundary value problems for differential equations with impulse action at non-fixed moments of time” (№AP 15473190, 2022-2024).

The personal contribution of the author. All results presented in the dissertation belong to the author himself. Co-authors and scientific consultants contributed to the formulation of the problem and discussion of the results.

Approbation of the received results. The main results of the work were presented and discussed at the following scientific events:

- traditional April international scientific conference, Institute of Mathematics and Mathematical Modeling of the Ministry of Education of the Republic of Kazakhstan (Almaty, Kazakhstan, April 5-8, 2020);
- IX International Scientific Conference “Differential Equations, Analysis and Problems of Algebra”, Aktobe Regional University named after. K. Zhubanova (Aktobe, Kazakhstan, May 24-28, 2022);
- traditional April international scientific conference, Institute of Mathematics and Mathematical Modeling of the Ministry of Education of the Republic of Kazakhstan (Almaty, Kazakhstan, April 15-20, 2024);
- Mathematical Analysis, Differential Equation & Applications – MADEA 9, Bishkek, Kyrgyzstan (June 21-25, 2021);
- International Workshop on the Qualitative Theory of Differential Equations QUALITDE – 2020, Tbilisi, Georgia (December19-21, 2020);

Publications. On the topic of the dissertation, 8 articles were published, including 2 publication in a ranking scientific journal indexed in the Scopus database, 2 publication in scientific journal included in the list recommended by the Committee for Quality Assurance in the Field of Science and Higher Education of the Ministry of Science and Higher Education of the Republic of Kazakhstan for publication of the main scientific results of scientific activities, 4 publications in the materials of the international scientific conferences.

Structure and scope of the thesis. The thesis consists of an introduction, 3 chapters (the first chapter consists of 6 sections, the second chapter consists of 3 sections), conclusion and a list of references.

Summary of the dissertation. The introduction includes an assessment of the current state of the issues under consideration and justification of the need for scientific research. The introduction shows the relevance and novelty of the topic, the main goals and objectives of the study, as well as the provisions submitted for defense.

The first part discusses the method of averaging the solution of boundary value problems for differential equations with the impulse action at non-fixed moments of time.

Subsection 1.1 presents the formulation of the boundary value problem for the differential equations under the impulse action at non-fixed moments of time, as well as the main and additional conditions and definitions for it.

Subsection 1.2 considers the continuous dependence of the solution to the system of the equations with the impulsive action on the initial conditions.

First of all, the continuous dependence of the solution of the system of the equations with the impulse action on the initial conditions is investigated, and then the continuous differentiability of the solution from the initial data for the impulse system at a fixed time is investigated.

Subsection 1.3 considers solving the variational equation of the impulsive system at a fixed time using the averaging method.

At a fixed time, the system of the differential equations with the impulse action was considered, the corresponding equation in variations was studied, and as a result, the theorem on averaging the equation in variations was formulated.

Subsection 1.4 considers the solution of a boundary value problem for the differential equation with the impulse action at fixed times.

The solvability of the boundary value problem for the differential equations with the impulse action at a fixed time is investigated, as a result, the theorem is formulated.

Subsection 1.5 considers the solution of a boundary value problem for the system of differential equations with the impulse action at non-fixed moments of time using the averaging method.

The problem of the solvability of the boundary value problem for the system of differential equations with the impulse action at an unfixed moment in time with a small parameter is investigated by averaging, and the main result is formulated in the form of the theorem.

Subsection 1.6 considers two-sided bounded solutions on the axis of the systems with the impulsive effects at non-fixed times.

The two-sided axis-limited solution of the system of the differential equations with the impulse action at an unfixed moment in time with a small parameter was studied first on a half-interval, then on the entire numerical axis, and the results were formulated in the form of the theorem.

Subsection 2.1 considers the solution of a boundary value problem for the nonlinear ordinary differential equation with the impulse action using the parameterization method.

First, the interval under consideration is divided into subintervals through the pulse points. Then the parameter is introduced as the value of the unknown function in the middle of the subintervals, the function is replaced in each subinterval, then the given boundary value problem is reduced to an equivalent multipoint problem with the parameters. An algorithm for the solving a multipoint boundary value problem with the parameters is constructed, the theorem is formulated and proven, which is a sufficient condition for the convergence of this algorithm, as well as the existence of an isolated solution to the multipoint boundary value problem.

Subsection 2.2 presents one modification of the algorithm for the solving the two-point boundary value problem for an ordinary differential equation.

Subsection 2.3 considers the solution of the boundary value problem for an ordinary differential equation with the impulse action at a non-fixed time using the parameterization method.

First, the parameters are introduced as the value of the unknown function at the two ends of the interval and the function is replaced at each subinterval, then the given boundary value problem is reduced to an equivalent multipoint problem with the parameters. An algorithm for the solving a multipoint boundary value problem with parameters is constructed, the theorem is formulated and proven, which is a sufficient condition for the convergence of this algorithm, as well as the existence of a unique solution to the multipoint boundary value problem.