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RESEARCH OF BOUNDARY VALUE PROBLEMS AND MULTIPERIODIC SOLUTIONS THE SYSTEM OF INTEGRO-DIFFERENTIAL EQUATIONS WITH AN OPERATOR OF DIFFERENTIATION WITH RESPECT TO DIRECTIONS OF VECTOR FIELD

ABSTRACT

of the thesis for the degree of Doctor of Philosophy (PhD)
in the specialty 6D060100 - Mathematics

Stucture and scope of the thesis. The thesis consists of an introduction, 2 chapters (the first chapter includes 4 sections and 6 subsections, the second chapter consists of 3 sections), a conclusion and a list of references.

The number of illustrations, tables and references. In the thesis used 136 references.

Keywords. Integro-differential, finite-hereditary, differentiation operator, multiperiodicity, matricant, convective-diffusion, parabolic type, initial-boundary value problem.

The actuality of the thesis. Phenomena whose mathematical models are described by integro-differential equations are common in biology, especially in conditions related to heredity. In engineering, similar phenomena are observed in the theory of elasticity. For example, if an elastic column is previously bent or formed, then its subsequent bending or twisting is very closely related to the previous position. Thus, it follows that hereditary conditions also exist in mechanics. Similarly, the subsequent state in electromagnetic fields is described, depending on their previous and current conditions.

Thus, it is known that in the world there are hereditary, that is, eredite phenomena characterized by the past and the present, along with determinant phenomena, the future of which is characterized not only by the present, but also by the past. The study of such phenomena is associated with the name of V. Volterra. The property of such phenomena that create fluctuations in time is very important. In biology, this is called fluctuation development. For example, if rabbits and wolves, fry and pike, or geese and pheasants live in places isolated from other animals, then with a decrease in food prey (rabbits, fry, geese), predators (wolves, pike, pheasants) begin to appear. The reduction of prey creates a condition for the growth of prey and ensures the growth of reduced predators. The growth of predators leads to a decrease in prey. The study of such temporal fluctuations in development is a very valuable, urgent problem. The connection between modern human development and the production of nutritious feed on Earth also comes from a group of fluctuations. The simplest such oscillations are periodic fluctuations. Periodic changes are dosed by frequencies. And if the changes occur under the action of several periodic forces, then complex oscillations occur, and the magnitude of the changes is determined by the

dimension of the frequencies. In this regard, multicomponent oscillatory multiperiodic phenomena arise. The fact that one of the periods of several accompanying periodic phenomena is not measured by others has led to the classification of time, i.e. the concept of multidimensional time. Thus, periodic phenomena in multidimensional time are called multiperiodic. Further, the definition of velocity in multidimensional time led to the concept of a differentiation operator in the directions of a vector field. So, the topic of the study is devoted to the study of an actual problem.

The dissertation work investigates the phenomenon describing the desired function $u(t,\tau)$ by $(t,\tau)=(t_1,...,t_m,\tau)$ multidimensional time variables $(\omega,\theta)=(\omega_1,...,\omega_m,\theta)$ periodic solutions of differential operator vector-matrix equations

$$D_c u(t,\tau)=A(t,\tau)u(t,\tau)+\int_{\tau-\varepsilon}^{\tau}K(t,\tau,\sigma,s)u(\sigma,s)ds+$$
$$+f\left(t,\tau,u(t,\tau),\int_{\tau-\varepsilon}^{\tau}K(t,\tau,\sigma,s)u(\sigma,s)ds\right) \qquad (0.1)$$

with the differentiation operator $D_c=\frac{\partial}{\partial\tau}+\left\langle c,\frac{\partial}{\partial t}\right\rangle\equiv\frac{\partial}{\partial\tau}+\sum_{j=1}^{m}c_j\frac{\partial}{\partial t_j}$.

Here $c=(c_1,...,c_m)$ – a constant vector defining the field of directions of the operator D_c; $A(t,\tau)$ – matrix operator describing the relationship of unknown variables $u=(u_1,...,u_m)$; $K(t,\tau,\sigma,s)$ – the core of the integral term that characterizes the hereditarity of the phenomenon; $\varepsilon>0$ – the number defining the heredite period, $f(t,\tau,u,v)$ – external exciting force, $\sigma=t-c\tau+cs$ – characteristic of the operator D_c. In the study , a variable is taken as a parameter for integrating the equation and the initial condition τ. A method for investigating a two-point boundary value problem for this variable for the system under consideration is proposed. This problem is a generalization of the problem of multiple periodicity of solutions.

In the future, in order to study the state of the diffusion nature of the phenomenon, the (ω,θ)- periodicity conditions are determined by (t,τ) solutions of boundary value problems for the integro-differential equation

$$D_c u(x,t,\tau)-a^2\frac{\partial^2 u(x,t,\tau)}{\partial x^2}+\chi\frac{\partial u(x,t,\tau)}{\partial x}=A(x,t,\tau)u(x,t,\tau)+$$
$$+\int_{\tau-\varepsilon}^{\tau}K(x,t,\tau,\sigma,s)u(x,\sigma,s)ds+f(x,t,\tau,u(x,t,\tau)) \qquad (0.2)$$

parabolic type with a positive component $c=(c_1,...,c_m)$ and proposed research method. Here $a>0$ and $\chi\geq 0$ – const; A,K and f – given matrix and vector

functions. If we consider the limiting case $\chi \to 0$, then the system (0.2) passes into the diffusion equation.

In the dissertation, multidimensional time is expressed by a vector $(\tau = t_0, t_1, ..., t_m) = (t, \tau)$, and the relationship between its dimensions $t = (t_1, ..., t_m)$ and τ defined with $c = (c_1, ..., c_m)$ – a constant vector field. Hence, the speed of the phenomenon is determined by the differentiation operator $D_c = \partial/\partial\tau + \sum_{j=1}^{m} c_j \partial/\partial t$, given above in this time field. In the special case, if $c_1 = ... = c_m = 1$, then we have the operator considered by V.Kh. Kharasakhal. The choice of such an operator is associated with a small generalization of the well-known operator, and such an operator is often used in KAM theory. The velocity of the phenomenon describing the vector $u = (u_1, ..., u_m)$ in the time field (t, τ) defined by the expression $D_c u$. This phenomenon is hereditary. Thus, its mathematical model together with the vector function $u = u(t, \tau)$ depends on the value given by the integral expression $\int_{s_0}^{\tau} F(t, \tau, \sigma, s, u(\sigma, s))ds$, characterizing hereditary. Here $F(t, \tau, \sigma, s, u)$ – known vector function, $\sigma = t - c\tau + cs$ – characteristic of the vector field, s_0 – a parameter that basically measures the time up to this point. Most often $s_0 = \tau_0$ – determines the initial moment of heredity, $s_0 = -\infty$ – the unlimited beginning of the phenomenon of hereditary to the present moment τ and moment $s_0 = \tau - \varepsilon$, expressing time $\varepsilon > 0$ the effects of heredity from the moment $\tau - \varepsilon$ to the present τ. In the case of fluctuating hereditary phenomena, the parameter may have values $s_0 = \infty$ or $s_0 = \tau - \varepsilon$. In the dissertation work, we limited ourselves to the consideration of latter case. In the considered systems , the ereditarian term was linearly determined by the integral $\int_{\tau-\varepsilon}^{\tau} K(t, \tau, \sigma, s)u(\sigma, s)ds$ or $\int_{\tau}^{\tau+\varepsilon} K(t, \tau, \sigma, s)u(\sigma, s)ds$. The study defines the conditions for the frequency of such phenomena and shows the ways to solve boundary value problems.

The relevance of the research tasks is also related to the applications of hereditary oscillations in mechanics, electromagnetism and biological phenomena. In many cases, especially in chemical reactions and hydrodynamics, hereditary phenomena are accompanied by diffusion phenomena. So, if a chemical mixture is sprayed into river water to clean mud, then an ereditar-diffusion phenomenon begins from this period. The beginning of the study of such phenomena is associated with G. Evans, a student of V. Volterra. The diffusion phenomenon is semi-determinant, its future is characterized only by the initial stage and it is irreversible. In this case, the diffusion of the problem considered in the dissertation work is determined by the operator $D_c u - a^2 \frac{\partial^2 u}{\partial x^2}$, the vector is a positive component. The study examines the

multiperiodicity of such phenomena. Modified methods of solving boundary value problems of parabolic type are also used for these phenomena.

Thus, the dissertation research is devoted to solving fundamental theoretical and priority topical problems related to applications of natural-technical processes.

The review of the scientific literature clarifying the relevance of the dissertation research begins with the works of V. Volterra and his followers in the field of integro-differential equations, which appeared at the beginning of the XX century. And the research of the post-Soviet period was carried out in the spirit of the works of A.I.Nekrasov, S.L.Sobolev, N.N.Nazarov, V.V.Vasiliev, V.S.Vladimirova, Ya.V.Bykov, M.I.Imanaliev and others. It is known that such studies are carried out first for linear equations and systems with one independent variable and are devoted to the initial boundary value problems of the work: L.E.Krivoshein, Yu.A.Ved, G.A.Shishkin and periodic solutions: A.B.Tkach, L.A.Talipova and others. Further developments and other areas of the theory of systems of integro-differential equations can be found in the works of A.I.Botashev, M.T.Adonts, E.A.Barbashin, A.N.Filatov, Yu.N.Rabotnov, A.A.Ilyushin, B.E.Pobedrya, A.M.Samoylenko. It is impossible not to mention the fundamental industry scientific-monographic and educational-methodical publications of Yu.A.Mitropolsky, N.N. Bogolyubov, V.I. Arnold, which were used in the course of this study. From a theoretical and methodological point of view, the research is based on the works of V.H.Kharasakhal, D.U.Umbetzhanov, Zh.A.Sartabanov, A.B.Berzhanov, G.A.Abdikalikov. Note also the works of K.-S,Chiu, T.K.Yuldasheva, S.A.Aisagalieva, which increased scientific interest, stimulated dissertation research. Here are studies 1) related to integro-differential equations, 2) in which periodic or almost periodic solutions of integro-differential equations with one independent variable are studied, 3) where boundary value problems for some integro-differential equations of parabolic type with two independent variables are considered.

The dissertation work presents the results obtained by the study of multi-frequency oscillatory solutions of the systems under consideration by the method proposed by V.Kh. Kharasakhal, which in the course of his research D.U. Umbetzhanov actually turned into a real method, further developed by Zh.A. Sartabanov and in the dissertation research extended to integro-differential systems that take into account the finite hereditary and diffusion of the described phenomena.

Let us pay attention to the fact that the proposed research methods are promising and can be applied to similar problems of related processes.

The aim of the thesis research is the study of initial problems for bounded hereditary quasilinear integro-differential systems of equations with a differentiation operator in the direction of a vector field, problems of multiperiodic solutions, the study of boundary value problems and their development for integro-differential equations of parabolic type.

The research problems:

a) determination of sufficient conditions for the existence of solutions to initial problems for bounded hereditary linear and quasilinear integro-differential systems of

equations with a differentiation operator for a given constant vector field of differentiation;

b) investigation of the existence and uniqueness of multiperiodic solutions of systems of hereditary linear and quasilinear integro-differential equations with a differentiation operator with respect to a vector field and their construction;

c) investigation of the solution of a two-point boundary value problem with respect to one of the time variables for linear and quasilinear systems with an finite-hereditary and operator of differentiation with respect to the vector field of systems of integro-differential equations;

d) investigation of multiperiodic time-variable solutions of boundary value problems for hereditary linear and quasilinear systems of integro-differential equations with a differentiation operator over a vector field of parabolic type.

The research methods. The well-known methods and results of the theory of partial differential equations, the theory of oscillations and the theory of operators are widely used in the dissertation work. The main method of research and solving the problems considered in the dissertation are the methods of Kharasakhal-Umbetzhanov and the methods of the works of Zh.A. Sartabanov on their development and generalization. In the dissertation work, a new method for investigating a two-point boundary value problem for systems of finite-hereditary integro-differential equations was proposed and tested for the first time.

The objects of research are solutions of initial, multiperiodic time variables and boundary value problems for linear and quasi-linear systems of integro-differential equations with bounded hereditarity and a differentiation operator over a vector field.

The scientific novelties:

1) the conditions for the multiperiodicity of the zeros of the differentiation operator D_c are established; the conditions of unique solvability of the initial problem of a linear homogeneous integro-differential equation with finite-hereditary equal to the period are determined; a resolving operator is constructed and a representation of the solution of the initial problem is found; conditions for the absence of multiperiodic solutions of the equation under consideration, except for zero, are indicated; sufficient conditions for the existence and uniqueness of a multiperiodic solution of a linear inhomogeneous integro-differential equation with finite-hereditary are obtained; conditions are established the existence of a matrix function of Green type;

2) in the case of an arbitrary period of hereditary, more general conditions for the multiperiodicity of the zeros of the differentiation operator are established; necessary and sufficient conditions for the periodicity of a homogeneous linear integro-differential equation with an arbitrary finite-hereditary; a representation of the solution of the initial problem of a linear inhomogeneous integro-differential equation with arbitrary finite-hereditary is derived; conditions for the existence of a Green type matrix function of the problem of multiperiodic solution of this equation are found and its integral representation with an estimate is given;

3) sufficient conditions for the existence and uniqueness of a multiperiodic solution of quasilinear systems of integro-differential equations with arbitrary finite-hereditary are indicated when the nonlinearities of the systems a) do not contain and b) contain an integral hereditary term with a finite period;

4) sufficient conditions for the unique solvability of a two-point boundary value problem for linear and quasilinear systems of integro-differential equations with a differentiation operator in the case of arbitrary finite-hereditary are obtained;

5) sufficient conditions for the unique solvability of the initial and two-point boundary value problem for a linear system of integro-differential equations of parabolic type with a differentiation operator with finite-hereditary are found; conditions for multiperiodicity in time variables of this solution of the system are given;

6) the conditions of unique solvability of the problem of multiperiodicity with respect to time variables and limitation with respect to a spatial variable along the semiaxis of solutions for linear systems of finite-hereditary integro-differential equations of parabolic type with a differentiation operator are established;

7) sufficient conditions are presented for the unique solvability of the problem of a multiperiodic solution with respect to time variables and a limited spatial variable for linear and quasi-linear systems of integro-differential equations of finite-hereditary and convective-diffusion type.

The results of the thesis which are taken out on defense:

– sufficient conditions for the existence and uniqueness of the solution of the initial problem for systems of integro-differential equations of a given finite-hereditary with a differentiation operator on the direction of a vector field based on the construction of a resolving operator and sufficient conditions for the existence and uniqueness of multiperiodic solutions of such systems with an integral representation of them in terms of a Green-type matrix function;

– necessary and sufficient conditions for the existence and uniqueness of multiperiodic solutions of linear systems of integro-differential equations of arbitrary finite-hereditary with the differentiation operator and their integral representations;

– extension of linear case methods to quasilinear systems of integro-differential equations of arbitrary finite-hereditary with a differentiation operator and sufficient conditions for the existence of multiperiodic solutions of the systems under consideration when the nonlinearities of the systems a) do not contain and b) contain an hereditary term;

– generalization of the method of multiperiodic solutions and sufficient conditions for the solvability of two-point boundary value problems for linear and quasilinear systems of integro-differential equations of finite-hereditary with a differentiation operator with respect to a vector field;

– method for investigating and establishing sufficient conditions for unique solvability of the initial and two-point boundary value problem for a linear system of parabolic type of finite-hereditary integro-differential equations;

– the condition for modifying one method of unique solvability of the problem of multiperiodicity in time variables and boundedness in space variable along the

semiaxis of the solution for a linear system of finite-hereditary parabolic integro-differential equations with a differentiation operator in a vector field;

– representation of the conditions for unique solvability of the problem of multiperiodicity in time variables and boundedness in spatial variable solutions for linear and quasilinear systems of finite-hereditary and convective-diffusion integro-differential equations with a differentiation operator in a vector field.

The personal contribution of the author. All the results given in the dissertation were obtained by the author independently. The participation of scientific consultants consists in setting the task of discussing the results obtained.

Approbation of the received results. The main results of the work were reported and discussed at the following conferences and seminars:

– VIII International Scientific Conference "Problems of Differential equations, analysis and Algebra". Aktobe, November 1, 2018;

– International conference "Mathematical analysis, Differential equation and applications (MADEA 2018) ". Bishkek: KTMU, 17-23 June, 2018;

– International scientific Conference "Theoretical and applied problems of mathematics, mechanics and computer science", dedicated to the 70th anniversary of Doctor of Physical and Mathematical Sciences, Professor M.I.Ramazanov. Karaganda, June 12-13, 2019;

– International Conference "Actual Problems of Analysis, Differential Equations and Algebra" (EMJ-2019) dedicated to the 10th anniversary of the issue of the Eurasian Mathematical Journal. Nur-Sultan, October 16-19, 2019;

– The traditional April International Mathematical conference in honor of the Day of Science Workers of the Republic of Kazakhstan. Almaty, April 5-8, 2020;

– Traditional April International Mathematical conference in honor of the Day of Science Workers of the Republic of Kazakhstan, dedicated to the 75th anniversary of Academician of the National Academy of Sciences of the Republic of Kazakhstan T.Sh.Kalmenov. Almaty, April 5-8, 2021;

– IX International Scientific Conference "Problems of Differential Equations, Analysis and Algebra". Aktobe, May 24-28, 2022;

– Scientific seminar "Research of nonlinear optimization problems of systems with distributed parameters". Kyrgyz-Russian Slavic University, Bishkek, Kyrgyzstan (seminar leader Doctor of Physical and Mathematical Sciences, professor A.Kerimbekov);

– Scientific seminar "Qualitative theory of differential Equations", J. Balasagyn Kyrgyz National University, Bishkek, Kyrgyzstan (seminar leaders Doctor of Physical and Mathematical Sciences, professor A.Saadabaev, Doctor of Physical and Mathematical Sciences, professor B.K. Temirov);

– Scientific seminar "Qualitative and approximate methods for the study of differential equations", Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan (seminar leader Doctor of Physical and Mathematical Sciences, professor A.T.Asanova);

– Scientific seminar "The Problems of Applied Mathematics and Computer Science", Department of Mathematics, K. Zhubanov Aktobe Regional University,

Aktobe, Kazakhstan (seminar leader Doctor of Physical and Mathematical Sciences, professor Zh. Sartabanov).

Publications. On the topic of the dissertation, 15 articles were published, including 1 publication in a ranking scientific journal indexed in the Scopus database, 4 publication in scientific journal included in the list recommended by the Committee for Quality Assurance in the Field of Science and Higher Education of the Ministry of Science and Higher Education of the Republic of Kazakhstan for publication of the main scientific results of scientific activities, 10 publications in the materials of the international scientific conferences.