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CONSTRUCTION OF SOLUTIONS OF THE WHITTAKER TYPE SYSTEMS NEAR SINGULAR CURVES

ABSTRACT

of the thesis for the degree of Doctor of Philosophy (PhD) in the specialty 6D060100 – Mathematics

Structure and scope of the thesis. The thesis consists of an introduction, 3 sections (the first section includes 3 subsections and 5 paragraphs, the second section consists of 5 subsections and 4 paragraphs, the third section consists of 2 subsections and 10 paragraphs), a conclusion and a list of references.

The number of illustrations, tables and references. In the thesis used 93 references.

Keywords. Rank, anti-rank, normal-regular solution, special curves, Frobenius-Latysheva method, Whittaker-type system, Horn-type system, Bessel-type system, degenerate hypergeometric function, Laguerre polynomials, Appell function, Lauricella series, Khudozhinikov function, Humbert function, system of defining equations.

The actuality of the thesis. The analytical theory of differential equations originates from the studies of O. Cauchy, L. Fuchs, G. Frobenius, K. Gauss, Y. Horn and etc. Fundamental research on the analytical theory of differential equations with variable coefficients was developed by L. Fuchs.

A. Poincaré, developing ideas from the works of L. Fuchs and L. Tome, investigated the problem of asymptotic decomposition of functions. Of considerable interest is the development of the Frobenius-Latysheva method on analytical systems of differential equations in second order partial derivatives, as well as the study of the behavior of the solution of the equations in the vicinity of special curves.

In the case of a function of one variable Lucy J. Slater notes that «degenerate hypergeometric functions are called four functions: Kummer functions ${}_1F_1(\alpha;\beta;x)$, associated second solution $U(\alpha,\beta;x)$ and two Whittaker functions $M_{k,m}(x)$ and $W_{k,m}(x)$».

Most of the functions used in mathematical physics, including the Weber and Bessel functions, are special cases of these functions. They are all regular partial solutions of some second-order differential equations. These equations have features: regular at a point $x=0$ and irregular at a point $x=\infty$. The Kummer equation and the Whittaker equation have such features.

Classification of features into regular and irregular allows the study of the behavior of decisions near these special points. In this work we use the classification given by K.Y. Latysheva with the help of the concept of rank introduced by A. Poincaré and the anti-rank introduced by L. Tome.

The relationship between the Kummer and Whittaker equations, as well as the related equations derived from them through various transformations, has been most studied. Having common properties with the above equations, related equations are widely applied in various problems of science and technology. Therefore, the literature devoted to the theory of ordinary differential equations gives a list of related equations with known applications of equations Kummer, Bessel, Whittaker, Laguerre, Jacobi, etc.

In the case of hypergeometric functions of two variables, the range of properties considered is extended. The study is complicated by the fact that if in the ordinary case there is only one degenerate hypergeometric equation, then in the case of two variables there are 20 degenerate hypergeometric functions. All of these are derived mainly from the four hypergeometric Appel functions $F_1 - F_4$ using limit transitions. Horn has established that they are private solutions to 20 systems of differential equations in second-order partial derivatives. Of these, as in the ordinary case, the systems connected with each other some common properties shall be called related. For example, a common property is a transformation where a polynomial $Q(x_1,...,x_n)$ from n variables can be derived from one system to another, and a relationship between their solutions as degenerate hypergeometric functions of two variables. The four hypergeometric functions $F_1 - F_4$ of P. Appel of the two variables, M. Lauricella generalized and introduced four hypergeometric F_A, F_B, F_C, F_D functions of the n variable. Furthermore, various of their properties and degenerate hypergeometric functions of the n variables, obtained by a limit transition were studied.

Various integral representations of hypergeometric functions have been introduced and investigated in connection with their important applications in various fields. The properties of hypergeometric functions are widely used in the study of degenerate equations.

It should be noted that the development of the Frobenius-Latysheva method for systems of differential equations in second order partial derivatives, which lead to various applied problems of electrodynamics, radioelectronics, mathematical physics, statistics, projective differential geometry, in various fields of theoretical physics.

In the theory of heat and mass transfer in electrical contacts to simulate heat exchange in bodies with variable cross-section, when the input data functions are analytical, the solution of the problem can be presented in the form of rows by special functions, Laguerre polynomial and degenerate hypergeometric function.

The relevance of the study is due, on the one side, to the need for in-depth study of degenerate hypergeometric systems of differential equations in second order partial derivatives, which are widely applied in the theory of special functions of many variables, as well as in applied problems of mathematical physics and multidimensional degenerate equations. On the other side, the need to establish different related systems such as Bessel, Laguerre, Whittaker and the study of the behavior of their decisions near special curves.

The types of basic degenerate systems are little known, such as Bessel-type systems and related systems. It is also not established that there exist systems whose solutions are orthogonal polynomials of two variables and which of the 20 degenerate systems they are related to. No common research method. For the study of such systems, a system consisting of two equations is applied generalized by Zh.N. Tasmambetov Frobenius-Latysheva method. It is necessary to generalize this method for the case of a system consisting of n differential equations in second-order partial derivatives and to develop the theory of constructing normal, normal-regular and finite solutions near specific curves.

The dissertation research is devoted to the study of the construction of solutions of related degenerate hypergeometric systems near special curves and the establishment of a number of new systems such as Bessel, Whittaker, Laguerre, as well as the relationship of their decisions among themselves.

In the study of degenerate hypergeometric systems, the system of differential equations in second order partial derivatives plays an important role

$$\left.\begin{array}{l} P^{(0)}(x,y)\cdot Z_{xx}+P^{(1)}(x,y)\cdot Z_{y}+P^{(2)}(x,y)\cdot Z=0 \\ Q^{(0)}(x,y)\cdot Z_{yy}+Q^{(1)}(x,y)\cdot Z_{x}+Q^{(2)}(x,y)\cdot Z=0 \end{array}\right\}, \qquad (0.1)$$

where the coefficients $P^{(i)}(x,y)$ and $Q^{(i)}(x,y)$ $i=\overline{0,2}$ analytical functions or polynomials of two variables.

It is related to the degenerate Horn system

$$x_j\frac{\partial^2 F}{\partial x_j^2}+(\gamma_j-x_j)\frac{\partial F}{\partial x_j}-\sum_{k\neq j}x_k\frac{\partial F}{\partial x_k}-\lambda F=0,\ \ j=\overline{1,n}, \qquad (0.2)$$

and with degenerate system

$$x_j\frac{\partial^2 U}{\partial x_j^2}-x_j\frac{\partial U}{\partial x_j}-\sum_{r\neq j}x_r\frac{\partial U}{\partial x_r}+\left[\frac{x_j^2}{4}-\frac{x_j}{2}\sum_{r\neq j}x_r+rx_j+\frac{1}{4}-\mu_j^2\right]U=0,\ \ j=\overline{1,n} \qquad (0.3)$$

obtained from (0.2) by transformation of the

$$F(x_1,...,x_n)=\exp Q(x_1,...,x_n)U(x_1,...,x_n), \qquad (0.4)$$

$$Q(x_1,...,x_n)=\frac{\alpha_{p0...0}}{p}x_1^p+...+\frac{\alpha_{0...0p}}{p}x_n^p+...+\alpha_{10...0}x_1+...+\alpha_{0...01}x_n\ , \qquad (0.5)$$

where $F=F(x_1,x_2,...,x_n)$ general unknown for all n system equations (0.2).

Uncertain coefficients $\alpha_{p,0,...,0},\alpha_{0,0,...,p},...,\alpha_{1,0,...,0},\alpha_{0,0,...,1}$ of polynomial (0.5) and coefficients of generalized power series are determined by Frobenius-Latysheva method. The studied related systems of Bessel, Laguerre and Whittaker are obtained from systems (0.1) to (0.3) by a limit transition and transformation (0.4).

The aim of the thesis research is to study in depth the degenerate hypergeometric systems of differential equations in second order partial derivatives, to establish a number of new systems such as Bessel, Laguerre, Whittaker and related to Horn systems, development of efficient algorithms for constructing their solutions near regular and irregular special curves. Study of the possibility of the existence of normal-regular solutions of degenerate hypergeometric systems, obtained from Lauricella systems by means of limit transitions.

The research problems:

a) establishment of a common related system, based on an analysis of the characteristics of all 20 known degenerate Horn systems consisting of two equations of second-order;

b) establishment of Bessel-type systems related to Horn and Whittaker systems and generalization of their common properties to degenerate hypergeometric systems consisting n equations of second-order;

c) to develop efficient algorithms for building normal, normal-regular and finish decisions of related systems;

d) to establish Laguerre-type systems related to Horn systems and investigate the relationship between regular and regular solutions;

e) to investigate the existence of necessary conditions of normal-regular solutions of degenerate hypergeometric systems obtained from the Lauricella (F_D) system;

f) to obtain new functions in the form of normally regular solutions of many variables as a solution of degenerate hypergeometric systems, establishing a connection with the functions of Horn, Humbert, Bessel, Laguerre and Khudozhnikov.

The research methods. Well-known methods and results of the analytical theory of special functions and second-order partial differential equations are widely used in the dissertation work. The main method of research and solving the problems considered in the dissertation is the Frobenius-Latysheva method and the methods of work of J.N. Tasmambetov on their development and generalization.

The objects of the study are the problems of establishing degenerate hypergeometric related systems of partial differential equations of the second order.

Scientific novelty of the research. The scientific novelty lies in the dissemination of the generalized Frobenius-Latysheva method for the study of degenerate hypergeometric systems of partial differential equations of the second order.

In order to substantiate and develop, as well as use the proposed generalized method to solve specific problems related to the construction of related systems, the results determining the scientific novelty are obtained:

a) the new properties of the Whittaker-type system have been established, its connection to other related systems has been found;

b) substantiated implementation of Kummer's theorems for the Horn-type system and their applications to the construction of solutions of other related systems and connection with Humbert functions;

c) an algorithm for constructing normally regular solutions of related systems such as Whittaker and Horn is proposed;

d) regular and normally regular solutions of the established series of Bessel-type systems related to the Horn and Whittaker systems are constructed;

e) the generalized Frobenius-Latysheva method is widespread and new properties of multidimensional finite solutions of established Laguerre-type systems are obtained;

f) the necessary conditions for the existence of normally regular and finite solutions of degenerate hypergeometric systems derived from the Lauricella (F_D) system are established;

g) new functions for systems of degenerate hypergeometric related systems in the form of normally regular solutions of many variables are obtained and their connection with the functions of Horn, Humbert, Bessel, Laguerre and Khudozhnikov $\Phi_{D,n}^{k,l}(0 \le k, l \le n, n-1 \le k \le n)$ is established.

The results of the thesis which are taken out on defense:

– generalization of Kummer's theorem on Horn-type systems and its application to construction of solutions of other related systems;

– features of the application of the generalized Frobenius-Latysheva method to the construction of the solution of related systems near various special curves;

– algorithms for constructing normal-regular solutions of Bessel-type systems taking into account regular and irregular features;

– the necessary conditions for normal and regular decisions of related systems;

– algorithm of final solutions of Laguerre's related systems and construction of new solutions;

– construction of solutions of Bessel-type systems obtained by transformation from related systems such as Whittaker and Horn; features of establishment of new related systems;

– establishing a connection between the functions of the Khudozhnikov $\Phi_{D,n}^{k,l}(0 \le k, l \le n, n-1 \le k \le n)$ and the normal-regular solutions of many variables of the degenerate Lauricella-type (F_D) system by the limit transition and the application of the generalized Frobenius-Latysheva method.

The personal contribution of the author. All the results given in the dissertation were obtained by the author independently. The participation of scientific consultants consists in setting the task of discussing the results obtained.

Approbation of the received results. The main results of the work were reported and discussed at the following conferences and seminars:

– VIII International Scientific Conference "Problems of Differential equations, analysis and Algebra". Aktobe, November 1, 2018;

– The traditional April international mathematical conference in honor of the Day of Science Workers of the Republic of Kazakhstan and the Workshop "Problems of modeling processes in electrical contacts", dedicated to the 80th anniversary of

Academician of the National Academy of Sciences of the Republic of Kazakhstan S.N. Kharin. Almaty, April 3-5, 2019;

– International scientific Conference "Theoretical and applied problems of mathematics, mechanics and computer science", dedicated to the 70th anniversary of Doctor of Physical and Mathematical Sciences, Professor M.I.Ramazanov. Karaganda, June 12-13, 2019;

– International Conference "Actual Problems of Analysis, Differential Equations and Algebra" (EMJ-2019) dedicated to the 10th anniversary of the issue of the Eurasian Mathematical Journal. Nur-Sultan, October 16-19, 2019;

– The traditional April international mathematical conference in honor of the Day of Science Workers of the Republic of Kazakhstan, dedicated to the 1150th anniversary of Abu Nasir al-Farabi and the 75th anniversary of the Institute of Mathematics and Mathematical Modeling. Almaty, April 5-8, 2020;

– Traditional April International Mathematical conference in honor of the Day of Science Workers of the Republic of Kazakhstan, dedicated to the 75th anniversary of Academician of the National Academy of Sciences of the Republic of Kazakhstan T.Sh.Kalmenov. Almaty, April 5-8, 2021;

– IX International Scientific Conference "Problems of Differential Equations, Analysis and Algebra". Aktobe, May 24-28, 2022;

– The International Conference "Computing and Information Technologies in Science, Technology and Education" (CITech-2022), dedicated to the 90th anniversary of the birth of Academician N.K. Nadirov and the 80th anniversary of Academician M. O. Otelbaev. Almaty, October 12-15, 2022;

– The traditional international April mathematical conference in honor of the Day of Science Workers of the Republic of Kazakhstan. Almaty, April 4-8, 2023;

– Scientific seminar "The Problems of Applied Mathematics and Computer Science", Department of Mathematics, K. Zhubanov Aktobe Regional University, Aktobe, Kazakhstan (seminar leader Doctor of Physical and Mathematical Sciences, professor Zh. Sartabanov).

Publications. On the topic of the dissertation, 16 articles were published, including 2 publication in a ranking scientific journal indexed in the Scopus database, 3 publication in scientific journal included in the list recommended by the Committee for Quality Assurance in the Field of Science and Higher Education of the Ministry of Science and Higher Education of the Republic of Kazakhstan for publication of the main scientific results of scientific activities, 11 publications in the materials of the international scientific conferences.