Review

of the Ph.D. thesis of Meirambek Mukash

"Qualitative properties and analysis of the solution of a boundary value problem for a differential equation with impulsive action"

Differential equations with impulse effects serve as mathematical models of many objects that, in the process of their evolution, are subject to the action of short-term forces, for example, in nonlinear mechanics, in automatic control, in the theory of oscillations and dynamic systems, etc. The presence of impulse effects can significantly complicate the behavior of the trajectories of such systems even for the case of simple differential equations, it is sufficient. Impulsive systems of various types were considered in the classical works of N.N. Bogolyubov, N.M. Krylov, E.A. Barbashin, A. Khalanay, D. Veksler, A.D. Myshkis, A.M. Samoilenko, N.A. Perestyuk, D. D. Bainov, etc. Subsequent research by many authors was devoted to the study of stability of solutions to differential equations with impulse action, the development of the theory of periodic and almost periodic solutions of impulsive systems, the study of invariant sets, the construction of asymptotic expansions using the Krylov-Bogolyubov-Mitropolsky small parameter method, averaging method, problems of optimal control theory, study of impulsive systems with random perturbations.

It is well known that studies of real processes based on idealized mathematical models often lead to differential equations with small parameters. Various asymptotic methods are widely used to study them. The choice of a specific asymptotic method depends on the structure of the differential equation describing the dynamics of the object. Due to numerous applications, averaging methods have been widely developed in nonlinear mechanics and oscillation theory. The mathematical justification of the averaging method for ordinary differential equations originates in the fundamental work of N.M. Krylov and N.N. Bogolyubov. A major role in the development of the averaging method for various classes of differential equations played the works of E.A. Grebenikov, Yu.A. Mitropolsky, N.N. Moiseev, N.A. Perestyuk, V.A. Plotnikov, A.M. Samoilenko, A.N. Filatov and others.

The generalization of the averaging method for the asymptotic integration of impulsive differential equations is of great theoretical and practical importance for the following reasons:

due to the complex structure of pulsed systems, qualitative research is associated with great difficulties, while the averaged system becomes pulseless;

the solution of the averaged system approximates the solution of the original system with any predetermined accuracy on an asymptotically large time interval.

In this dissertation, the averaging method is developed for initial and boundary value problems for nonlinear differential equations with impulse effects at non-fixed moments and depending on a small parameter.

Differential equations with impulse actions are studied at non-fixed moments with a small parameter in the nonlinear right-hand side of the equation and under impulse conditions, where the boundary conditions are also specified nonlinearly.

The following questions are investigated: i) continuous dependence of the solution of a system of differential equations with impulse effects on the initial conditions; ii) solving the variational equation of the impulsive system by the averaging method; iii) conditions for the solvability of a boundary value problem for a differential equation with impulse effects at fixed times based on the averaging method; iv) conditions for the solvability of a nonlinear boundary value problem for a differential equation with impulse effects at non-fixed moments of time; v) conditions for the solvability of a boundary value problem for a differential equation with impulse effects at fixed times based on a modification of Dzhumabaev's parameterization method; vi) conditions for the solvability of a boundary value problem for a differential equation with impulse effects at a non-fixed moment in time using the parameterization method.

Thus, the main methods for solving these classes of problems are the averaging method and the Dzhumabaev's parameterization method.

An essential point of the study is the nonlinearity of the considered boundary value problems for differential equations with impulse actions at non-fixed moments and the dependence on the small parameter of the right side of the equation and the impulse conditions.

The thesis consists of two sections. The first section is devoted to the study of initial and boundary value problems for nonlinear differential equations with impulse effects at non-fixed moments and depending on a small parameter by the averaging method. The continuous dependence of the solution of the system of differential equations with impulse effects on the initial conditions is shown. The solution of the variational equation of the pulse system was constructed using the averaging method. Conditions for the solvability of the boundary value problem for the differential equation with impulse influences at fixed times are established using the averaging method. The properties of the boundary value problem for an averaged system make it possible to obtain conditions for the solvability of the original problem. Conditions for the solvability of the nonlinear boundary value problem for the differential equation with impulse effects at non-fixed moments of time are obtained.

In the second section, the boundary value problem for the differential equation with impulse effects is studied by the Dzhumabaev's parameterization method.

Conditions for the solvability of the boundary value problem for the differential equation with impulse effects at fixed times are established based on a

modification of Dzhumabaev's parameterization method. Conditions for the solvability of the boundary value problem for the differential equation with impulse effects at a non-fixed moment in time are obtained using the parameterization method.

The main results of the thesis are formulated in the form of theorems, which are provided with rigorous proofs. They have been published in high ranking scientific journals and approved at a number of international mathematical conferences and seminars.

In my opinion, this thesis fulfills the requirements for the doctoral dissertation. I recommend that Meirambek Mukash be awarded the degree of Doctor of Philosophy.

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