**Aktobe Regional University named after K. Zhubanov**
**Questions of Doctoral studies entrance exam on educational program**
**8D05401 - Mathematics**

1.Existence and uniqueness theorem for the solution of the Cauchy problem for an ordinary differential equation of the first order.
2.Linear differential equations of the n-th order with constant coefficients.
3.Linear differential equations of the n-th order with variable coefficients.
4.Dynamic systems and their research on the phase plane.
5.Stability of solutions of linear systems of differential equations.
6.The Cauchy-Kovalevskaya theorem for linear partial differential equations.
7. Symmetric non-negative linear operators. Eigenvalue problems for the operator of the second derivative.
8.Problems of Cauchy and Gursa for a general linear hyperbolic equation.
9. Equations of mixed type. The Tricomi problem for the Lavrent'ev-Bitsadze equation.
10. Generalized solution of the first initial-boundary value problem for an equation of parabolic type.
11. The curvature of a curve on surface.
12.The normal cross-section of surface. Meunier's theorem.
13.Methods for calculating the main directions and main curvatures at a given point of surface.
14.Geodesic lines. Theorem on the existence of geodesic lines on a regular surface.
15.Family of lines, envelope.
16. A system of linear algebraic equations. The Kronecker-Capelli theorem on the compatibility of a system of linear equations.
17. Laplace's theorem on the expansion of the determinant over several rows or columns.
18. Fundamental theorem of algebra of complex numbers.
19. Characteristic roots of the linear transformation and eigenvalues.
20. Sturm's theorem on calculating the roots of a polynomial.
21. Reduction to the canonical form of the $\lambda$ (lambda)-matrix.
22.A necessary and sufficient condition for the reducibility of a matrix to diagonal form.
23.Random variables, basic laws of distribution.
24.The probability distribution functions of a random variable.
25. Continuously differentiable functions, fundamental theorems about them. Uniform continuity. Cantor's theorem.
26.Limit points, upper and lower limits of the sequence. Cauchy criterion for the existence of a limit of function.
27.Properties of a definite integral. Estimates of integrals. Mean value theorems.
28.Improper integrals, convergence tests. The principal value of the improper integral.

29.Functions of bounded variation, their criterion. The Stieltjes integral, its properties.
30.Directional derivative of a function. Gradient. The Hamilton operator, its properties.
31. Sufficient conditions for the local extremum of functions of several variables.
32. Analytical functions. Cauchy-Riemann conditions. Properties of analytical functions.
33. The integral of a function of a complex variable. Cauchy's theorem. The Cauchy integral formula.
34.Absolute and conditional convergence of series. Signs of absolute convergence. Properties of convergent series.
35. Uniform convergence of functional sequences and series. Signs of uniform convergence. Properties of uniformly convergent series.
36. Laurent series. Isolated singular points of an analytic function.
37. Residue of the function relative to the singular point and its calculation.
38. Definition and examples of complete metric spaces. Continuous maps of metric spaces.
39. Definition and examples of normed spaces. Subspaces. Factor-space.
40. Hilbert space. The isomorphism theorem.
41. Linear functionals on normed spaces. Conjugate space. Examples.
42. Linear operators, their continuity, compactness.
43. Inverse operator, invertibility.
44. Measurable functions and their properties. Almost everywhere convergence. Convergence in measure.
45. Definition of the Lebesgue integral on a set of finite measure. Limit transition under the sign of the Lebesgue integral.
46. Implicit functions. Existence, continuity, differentiability of implicit functions.
47. Operator norm. Functional norm.
48. Spectrum of an operator. Resolvent.
49. Power series in the real and complex domain. The radius of convergence. Properties of power series.
50. Fourier series. Sufficient conditions for the representability of a function by a Fourier series.
51. Construction of a fundamental solution to a homogeneous differential equation with constant coefficients of the n-th order.
52. Using the Euler method, construct a solution to a linear system of differential equations with constant coefficients.
53. Integration of a linear system of differential equations with constant coefficients by the method of variation of arbitrary constants.
54. Construct the solution of the differential equation by the method of undefined coefficients.
55. Construct a solution to an inhomogeneous system by reducing a system of n linear equations to one equation of the n-th order.
56. Integration of differential equations using power series.
57. Matrix method of integration of linear systems of differential equations.

58. Continuous dependence of the solution of a normal system of differential equations on the initial data and parameters.
59. Using the phase plane method, construct a phase portrait of an autonomous system of the second order.
60. Investigation of stability by the method of Lyapunov functions.
61.Solve the Cauchy problem for the two-dimensional wave equation by the descent method.
62. Construct the solution of the Cauchy and Gursa problems for an equation of hyperbolic type by the Riemann method.
63. Solve the initial-boundary value problem for the parabolic equation by the method of separation of variables.
64. Construct the Green's function of the initial-boundary value problem for an equation of parabolic type.
65. Using the continuation method, construct a solution to the boundary value problem for the diffusion / heat conduction equation on the semi axis.
66. Apply the Riemann method to find a solution to the Cauchy problem of the telegraph equation.
67. Construct a solution to the Dirichlet problem for the Poisson equation by Green's method.
68.Construct the solution of the Neumann problem for the Poisson equation by Green's method.
69.Using the potential theory method, solve the first boundary value problem for the Laplace equation in a half-space.
70. Using the method of energy integrals, construct a solution to the mixed problem for an equation of hyperbolic type.
71. The asymptotic lines of surface. Properties of asymptotic lines.
72.The first and second quadratic forms of the surface of rotation.
73.Surfaces of constant curvature.
74. The contact of curves.
75. Equation of a line on plane. Parametric representation of the line.
76.Equation of a line in different coordinate systems.
77.Two types of tasks related to the analytical representation of the line.
78.The evolute of a plane curve.
79.Applications of the Taylor (Maclaurin) formula with various forms of residual terms.
80. The method of indeterminate Lagrange multipliers studies of functions on a conditional extremum.
81. Inequalities for sums and integrals (Jung, Helder, Minkowski).
82. Reducing a multiple integral to integrals by individual variables.
83. Calculation of integrals (proper and improper) that depend on the parameter.
84. Application of line integrals in vector analysis. Basic differential operations of vector analysis in curvilinear coordinates.
85. Theorems on residues and their application to the calculation of contour integrals.
86. Analytical continuation of the function. The Uniqueness theorem.

87. The principle of compressive mappings and its applications.
88. Compactness in metric spaces.Arcel's theorem.
89. The nested sphere theorem. Baer's theorem. Completion of space.
90.Convex sets and convex functionals. The Hahn-Banach theorem.
91. Decomposition of square-summable functions in a series by orthogonal systems.
92. Fourier transform, properties and applications.
93. Self-adjoint operators in a Hilbert space and their properties.
94. Recovering a function by its derivative. Absolutely continuous functions, their properties.
95. Bounded linear operators. Equivalence of the concepts of linear continuous and linear bounded operators.
96. Differential operators. Integral operators in spaces of functions.
97. Solve systems of linear equations by method of sequential elimination of unknowns (or by the Gauss method)
98. Determining the common roots of two polynomials by the Euclidean algorithm.
99. Reducibility of matrices to the canonical form.
100.Reducibility of matrices to Jordan normal form.
101. Reduction of the Cauchy problem for a linear differential equation to the Volterra integral equation and its solvability.
102. Invariance of a linear differential equation with respect to any transformation of the independent variable and with respect to a linear transformation of the desired function.
103. Efficiency of application of the method of successive approximations (Picard's method) in the research of the problem of existence and uniqueness of the initial problem for some differential equations.
104. The structure of the fundamental system of solutions of a homogeneous linear system with constant coefficients and the influence on the structure of elementary divisors of the matrix of coefficients of the system.
105.Analysis of the behavior of second-order dynamical systems on the phase plane.
106. The connection between the autonomous system and the corresponding system in a symmetric form.
107. Criterion of stability in the first approximation.
108. Oscillatory character of solutions of linear homogeneous equations of the second order.
109. Boundary value problems for an ordinary differential equation of the second order and their physical content.
110. The Cauchy problem for a linear partial differential equation of the first order.
111. Well-posed of problem statement of mathematical physics. Examples of ill-posed boundary value problems.
112. Construction of a system of eigenfunctions, completeness of orthogonal systems of functions in various functional spaces.
113. Reducibility of the Sturm-Liouville problem to an integral equation.

114. Uniqueness and stability of the solution to the first boundary value problem for an equation of parabolic type.
115. Construction of eigenvalues and eigenfunctions of the Laplace operator in a circle.
116. Apply potential theory to reduce boundary value problems to integral equations: The Dirichlet problem for the Laplace equation.
117. Apply potential theory to reduce boundary value problems to integral equations: The Neumann problem for the Laplace equation.
118. Using the Tricomi method, prove the uniqueness of the solution of the T-problem for the Lavrent'ev-Bitsadze equation.
119. Application of difference methods for solving problems of mathematical physics: Solution of a mixed problem for the diffusion equation by the method of finite differences.
120. Application of difference methods for solving problems of mathematical physics: Solution of the Dirichlet problem for Poisson's equation in a rectangle by the method of finite differences.
121. Semi-geodesic coordinate systems.
122.Basic equations of the theory of surfaces.
123.Investigation of the shape of second-order surfaces by their canonical equations.
124.Average curvature. Minimal surfaces.
125. Full curvature. Surfaces of constant negative curvature.
126. Theorems on implicit functions and their applications.
127. Relationship between Volterra integral equations and linear differential equations.
128. Application of contraction mapping principle to systems of linear algebraic equations.
129. Application of contraction mapping principle in the theory of differential equations.
130. Application of the method of finding a fixed point of mapping a metric space into itself for constructing solutions to nonlinear ordinary differential equations.
131.Application of contraction mapping principle to integral equations.
132.Application of the Fourier transform to the solution of differential equations.
133. Basic integral formulas of analysis and their applications. Green's Formulas.
134. Generalized functions. Fundamental solutions of linear differential operators with constant coefficients.
135. Applications of power series theory.
136. Gradient method for finding extremums of strongly convex functions.
137.Harmonic functions and their properties. Application of harmonic functions in mathematical physics.
138. Application of Fourier series in solving boundary value problems of mathematical physics.
139. Solving variational problems with fixed ends. Particular cases of the Euler equation.
140. Conformal mappings and examples of their application.

141. Applications of the matrix rank calculation method in solving vector algebra problems.
142.Comparative analysis of methods for calculating the rank of a matrix.
143.Comparative analysis of the Euclid algorithm and the Gorner method.
144.Application of the basic theorem of the algebra of complex numbers in mathematical analysis and algebra.
145.Finding the parameters of the sample equation of the straight line of the root-mean-square regression from ungrouped data.
146.Finding the parameters of a sample equation of a straight regression line from grouped data.
147.Method for calculating the sample correlation coefficient.
148.Testing the hypothesis of the normal distribution of the general population. Pearson's criterion of agreement
149.Sampling Spearman's rank correlation coefficient and testing the hypothesis of its significance.
150.The integral of a random function and its characteristics.

## References


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3.Треногин В.А. Обыкновенные дифференциальные уравнения. М.: ФИЗМАТЛИТ, 2009. -250с.

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- М.: ФИЗМАТЛИТ, 2008. -144 с.

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9. Бицадзе А.В. Уравнения математической физики. М.: Наука, 1996. -336 с.

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11. Э.Г.Позняк, Е.В.Шикин. Дифференциальная геометрия. М.: Изд-во Моск.Ун-та, 1990. - 384с.

12. Курош А.Г. Курс высшей алгебры. М.: Наука, 1968. - 431 с.

13. Гмурман В.Е. Теория вероятностей и математическая статистика. М.: Высшая школа, 2000. - 479 с.

14. В. А. Ильин, В. А. Садовничий, Бл. Х. Сендов. Под ред. А. Н. Тихонова. Математический анализ. Начальный курс. М.: МГУ, 1985. – 662 с.

15. Натансон И.П., Теория функций вещественной переменной, М.: Наука, 1974. – 480 с.

16. Фихтенгольц Г.М. Курс дифференциального и интегрального исчисления. Т3. 1966. – 662 с.

17. Привалов И.И. Введение в теорию функций комплексного переменного. – СПб.: Изд– во «Лань», 2009. – 432 с.

18. В. А. Ильин, В. А. Садовничий, Бл. Х. Сендов. Под ред. А. Н. Тихонова. Математический анализ. Продолжение курса. М.: МГУ, 1987. – 358 с.

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21. Демидович Б.П. Лекции по математической теории устойчивости. М.: Наука, 1997. -472с.

22. Владимиров В.С., Жаринов В.В. Уравнения математической физики:
- М.: ФИЗМАТЛИТ, 2008. -144 с.

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31. Тихонов А.Н., Самарский А.А. Уравнения математической физики. – М.: Изд - во МГУ, 1999. – 799 с.
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