Project name, IRN	AP25794821 - An Efficient Method for Finding Global Minima of Multimodal Functions and Its Applications
Completion date	27.02.2025-31.12.2027
Project supervisor	Tutkusheva Zhailan Salavatovna
Report	In the modern world, in all areas of human activity, there are tasks that require calculating the best or worst values of a certain quantity. There is a need to solve global optimization problems in technical, economic, engineering, biological, and other applied fields. Currently, machine learning and artificial intelligence are rapidly developing, where global optimization plays a key role, as it determines how quickly and efficiently the model finds the best parameters to minimize the loss function. In neural network training, several main optimization methods are distinguished, which affect the quality and speed of training. The most common optimization methods in the field of artificial intelligence and machine learning are gradient descent and its variations. However, they cannot overcome the problems of local minima and slow convergence. The search for the global minimum of the target function is a complex task, and there are no sufficiently universal methods for its solution. Analytical methods are inapplicable multi-extremal functions of large dimensions and usually lead to very complex and non-guaranteed results. Therefore, the task of global optimization is considered not fully studied and requires constructive methods. The proposed project leader has developed a method based on a special auxiliary function, and this method has been used to defend a dissertation. The developed method is suitable for optimizing multimodal and multivariable functions. It is also important to note the simplicity of the algorithm and the method's convergence to a global minimum, whereas the most common method in machine learning, the gradient method, converges to a local minimum. During the development of the method, all obtained theoretical results were rigorously proven and published in various high-ranking scientific journals [1-11], and computational experiments were conducted, yielding the precise values of the desired minimum and its point coordinates. The current project is a continuation of the research on the global optimization method, where an auxiliary function plays an important role. This function is constructed by transforming the multivariable objective function into a special single-variable function. The project will thoroughly study important properties of the new function and develop numerical implementation programs for the method. The goal of this project is to continue the deep and broad exploration of the developed method for various classes of functions, based on a special auxiliary function, to implement the numerical realization of this method, and apply it to solving various current applied problems in machine learning and artificial intelligence. The project plans to apply the developed method in machine learning and/or artificial intelligence (AI) training, as well as to numerically implement the method for specific tasks. Undoubtedly, research on such a pressing issue as the optimization of AI training has

	practical significance for the scientific and technological development of the Republic of Kazakhstan.
Purpose	The aim of the project is to continue a deep and comprehensive study of the possibilities of applying the developed method, based on a special auxiliary function, for various classes of functions, to conduct effective numerical implementations and to apply it to solve various current applied problems.
Expected results	At least 2 (two) articles will be published in journals ranked in the top three quartiles by impact factor in the Web of Science database or having a percentile of at least 50 in the CiteScore database of Scopus.
Research group	*Leader:* Zhailan Salavatovna Tutkusheva, Doctor of Philosophy (PhD) in Mathematics, Senior Lecturer (ORCID: 0000-0003-3611-9620, Scopus Author ID: 57457847600). *Scientific Consultant:* Khasen Toktaruly Otarov, Candidate of Physical and Mathematical Sciences, Associate Professor (H-index: 1, ORCID: 0009-0009-4471-1625, Scopus Author ID: 59562990800).
List of published works	1. Tutkusheva Zh., Otarov Kh.T. Application of the Auxiliary Function Method to the Search for the Global Minimum of Functions of Many Variables // Mathematical Modelling of Engineering Problems. – 2024. – Vol. 118, №5. – P. 1323-1329. https://doi.org/10.18280/mmep.110523 2. Kaidasov, Zh, Tutkusheva Zh. Algorithm for Calculating the Global Minimum of a Smooth Function of Several Variables // Mathematical Modelling of Engineering Problems. – 2021. – Vol. 8,№4. – P. 591-596. https://doi.org/10.18280/mmep.080412 3. Tutkusheva Zh.S., Kazbekova G. N., Seilkhanova R.B., Kairakbaev A. K. Wegstein's method for calculating the global extremum // Mathematical Modelling of Engineering Problems. – 2022. – Vol.9, №2. – P. 405-410. https://doi.org/10.18280/mmep.090214 4. Ramazanov M. D., Kaidasov Zh, Tutkusheva Zh. Studying the effectiveness of a new algorithm with a defining function for finding the global minimum of a smooth function // Известия НАН РК. Серия физико-математическая. - 2020. – Vol.332, №4. – P. 95-102. https://doi.org/10.32014/2020.2518-1726.70 **5.** Рамазанов М.Д., Туткушева Ж.С. Вычисление координат глобального минимума произвольной гладкой функции нескольких переменных // Вестник КазНИТУ. Физико-математические науки. - 2020. - №3(139). – С. 662-666. https://vestnik.satbayev.university/index.php/journal/issue/view/61/58 **6.** Туткушева Ж.С. Применение метода деления отрезка пополам в глобальной оптимизации на основе вспомогательной функции // Вестник КазНПУ им. Абая. Физико-математические науки. – 2022. – Т.79, №3. – С. 591-596. https://doi.org/10.51889/7099.2022.19.82.005 7. Кайракбаев А.К, Туткушева Ж.С. О свойствах одной вспомогательной функции для вычисления глобального

	экстремума // Вестник НИА РК №1 2024. – Т.91, №1. – С. 178-188. https://doi.org/10.47533/2024.1606-146X.17 8. Туткушева Ж.С. Определение координат глобального минимума произвольной гладкой функции // Современные инновации. IX международная заочная научно-практическая конференция «Современные инновации в эпоху глобализации: теория, методология, практика» (19-20 август 2019 г.). – 2019. – №4(32). – С. 5-7. 9. Туткушева Ж.С. Теоремы о сходимости нового метода глобальной оптимизации // Тезисы докладов Традиционной международной апрельской научной конференции. – Алматы. – 2022. – С. 113-115. 10. Туткушева Ж.С. Алгоритм глобальной оптимизации гладких функций нескольких переменных // Тезисы докладов Традиционной международной апрельской научной конференции. – Алматы. – 2021. – С. 63-64. Туткушева Ж.С. Свойства вспомогательной функции глобальной оптимизиции // IX Международная научная конференция «Проблемы дифференциальных уравнений, анализа и алгебры». – Актобе. – 2022. – С. 243-250.